

# Stable counteralignment of a circumbinary disc

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## ABSTRACT

In general, when gas accretes on to a supermassive black hole binary it is likely to have no prior knowledge of the binary angular momentum. Therefore a circumbinary disc forms with a random inclination angle  $\theta$  to the binary. It is known that for  $\theta < 90^\circ$  the disc will coalign wrt the binary. If  $\theta > 90^\circ$  the disc wholly counteraligns if it satisfies  $\cos \theta < -J_d/2J_b$ , where  $J_d$  and  $J_b$  are the magnitudes of the disc and binary angular momentum vectors respectively. If however  $\theta > 90^\circ$  and this criterion is not satisfied the same disc may counteralign its inner regions and, on longer timescales, coalign its outer regions. I show that for typical disc parameters, describing an accretion event on to a supermassive black hole binary, a misaligned circumbinary disc is likely to wholly co- or counter-align with the binary plane. This is because the binary angular momentum dominates the disc angular momentum. However with extreme parameters (binary mass ratio  $M_2/M_1 \ll 1$  or binary eccentricity  $e \sim 1$ ) the same disc may simultaneously co- and counter-align. It is known that coplanar prograde circumbinary discs are stable. I show that coplanar retrograde circumbinary discs are also stable. A chaotic accretion event on to an SMBH binary will therefore result in a coplanar circumbinary disc that is either prograde or retrograde with respect to the binary plane.

**Key words:** accretion, accretion discs – black hole physics – galaxies: evolution

## 1 INTRODUCTION

Galaxy mergers are commonly thought to be the main mechanism driving the coevolution of galaxies and their central supermassive black holes (SMBH). An SMBH binary is likely to form in the centre of the merged galaxy and subsequently accrete from a circumbinary disc. It is reasonable to expect that the angular momentum of the binary and that of the accreting gas are uncorrelated, and that each has no prior knowledge of the other. We can therefore expect a random distribution of orientations for such circumbinary discs.

Understanding the evolution of a misaligned circumbinary disc is needed if we are to understand the evolution of SMBH binaries. In a recent paper (Nixon et al., 2011) we showed that a retrograde circumbinary disc can be very efficient in extracting angular momentum from the binary orbit. This may offer a solution to the final parsec problem (Begelman et al. 1980; Milosavljević & Merritt 2001).

In another recent paper (Nixon, King & Pringle, 2011, hereafter NKP) we showed that the dominant effect of the binary potential, on a misaligned circumbinary disc, is to induce radially-dependent precessions of the misaligned disc particle orbits. This differential precession is known to induce warping of the disc as rings of gas dissipate energy through viscosity. The precession vanishes only when the orbit is in the plane of the binary (either prograde or retrograde) and thus we expect that both prograde and ret-

rograde orbits are possible equilibria for the gas. NKP showed that the evolution of a misaligned circumbinary disc is formally similar to the evolution of a misaligned disc around a spinning compact object. In this case the evolution is driven by the Lense–Thirring effect (e.g. Bardeen & Petterson 1975; Pringle 1992; Scheuer & Feiler 1996; Lodato & Pringle 2006; Nixon & King 2012). This implies that the analysis of King et al. (2005), which calculates the conditions for co- or counter-alignment, holds for circumbinary discs as well. The disc co- or counter-aligns depending on the magnitudes and directions of  $J_b$  and  $J_d$ , the angular momentum of the binary and disc respectively. The whole disc counteraligns with the binary if the initial inclination angle of the disc,  $\theta$ , and the magnitudes of the disc and binary angular momentum,  $J_d$  and  $J_b$ , respectively, satisfy

$$\cos \theta < -\frac{J_d}{2J_b}. \quad (1)$$

NKP only considered the zero-frequency (azimuthally symmetric  $m = 0$ ) term in the binary potential. This is a reasonable approach as all other terms induce oscillatory effects which cancel out on long timescales.

It is natural to assume that coaligned discs are stable, however in the past counteraligned discs have been incorrectly found to be unstable (Scheuer & Feiler, 1996). I therefore use a full three dimensional hydrodynamic approach to check the assumptions in NKP are valid and to confirm that co- and counter-alignment are stable. In section 2 I discuss the possibility of both co- and counter-alignment of the same disc (cf Lodato & Pringle 2006). In section 3

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I report a simulation of a counteraligning disc and give my conclusions in section 4.

## 2 SIMULTANEOUS CO- AND COUNTER-ALIGNMENT?

Lodato & Pringle (2006) considered the alignment of a disc and a spinning black hole. They showed that for a disc where  $\theta > \pi/2$  and  $J_d > 2J_h$ , initial counteralignment of the inner disc occurs, followed by subsequent coalignment of the outer disc. During the alignment of the outer disc the inner disc retains its retrograde nature and so a warp of significant amplitude is achieved ( $\Delta\theta \sim \pi$ ). This scenario produces a disc which is simultaneously counter-aligned (in the inner parts) and coaligned (in the outer parts). For the subsequent evolution of such a disc see Nixon, King & Price (2012).

If however the angular momenta are such that  $J_d \lesssim J_h$  then the disc can only wholly co- or counter-align and the above process is impossible. In this section we discuss whether it is feasible to have circumbinary discs such that both co- and counter-alignment are simultaneously possible. This possibility is constrained as the mass of the disc is limited by self-gravitational collapse and the radius of the disc is limited by the gravitational sphere of influence of the binary. Therefore there must be a maximum feasible disc angular momentum.

The disc simultaneously co- and counter-aligns if (and only if)  $\theta > \pi/2$  and the condition (1) does not hold. So for randomly aligned discs, simultaneous co- and counter-alignment is only possible if  $J_d \gtrsim 2J_b$  (assuming  $\cos \theta \sim -1$ ). Thus I derive the condition for  $J_d \gtrsim 2J_b$ . The disc angular momentum is

$$J_d \sim M_d \sqrt{GMR_d} \quad (2)$$

where  $M_d$  is the mass in the disc,  $M$  is the total binary mass,  $R_d$  is a characteristic radius for the disc and  $G$  is the gravitational constant. We note that the definition of  $J_d$  in this case is simply the total angular momentum in the disc. The exact definition of  $J_d$  is usually more subtle because the disc takes time to communicate its angular momentum.

The angular momentum of a binary with eccentricity  $e$  is

$$J_b = \mu \sqrt{GMa(1-e^2)} \quad (3)$$

where  $M$  is the total mass of the binary and  $\mu$  is the reduced mass ( $\mu \sim M_2$  for  $M_2 \ll M_1$ ).

So for simultaneous co- and counter-alignment we require

$$M_d \sqrt{GMR_d} \gtrsim 2\mu \sqrt{GMa(1-e^2)}. \quad (4)$$

Self-gravity limits  $M_d \lesssim (H/R)M$ , so we get

$$(H/R)M \sqrt{GMR_d} \gtrsim 2\mu \sqrt{GMa(1-e^2)}. \quad (5)$$

After a bit of algebra this tells us that we can only get simultaneous co- and counter-alignment if

$$\frac{R_d}{a} \gtrsim 4 \left( \frac{R}{H} \right)^2 \left( \frac{M_2}{M} \right)^2 (1-e^2) \quad (6)$$

For the most optimistic parameters we have  $a \sim 0.1$  pc and the sphere of influence of the binary  $\sim 10$  pc. This gives the LHS as  $\lesssim 100$ . For typical disc thickness ( $H/R \sim 10^{-3}$ ) it is clear that binaries with a low eccentricity ( $e \approx 0$ ) we cannot get simultaneous co- and counter-alignment unless the mass ratio is extreme ( $M_2/M_1 \lesssim 10^{-2}$ ). However if the disc is very thick then this is possible. For small binary mass-ratios the disc mass may be greater

than the mass of the secondary, therefore the hydrodynamic drag on the binary may become significant (Ivanov et al., 1999). If the binary is significantly eccentric, as predicted for binaries that have accreted through a retrograde disc (Nixon et al., 2011) then simultaneous co- and counter-alignment may be possible. I shall return to these possibilities in future work. However these arguments suggest that it is reasonable to assume an SMBH binary must wholly co- or counter-align the disc with the binary plane.

## 3 SIMULATION

To confirm the stable counteralignment of a circumbinary disc I perform one simple simulation. I use the SPH code PHANTOM, a low-memory, highly efficient SPH code optimised for the study of non-self-gravitating problems. This code has performed well in related simulations. For example Lodato & Price (2010) simulated warped accretion discs and found excellent agreement with the analytical work of Ogilvie (1999) on the nature of the internal accretion disc torques.

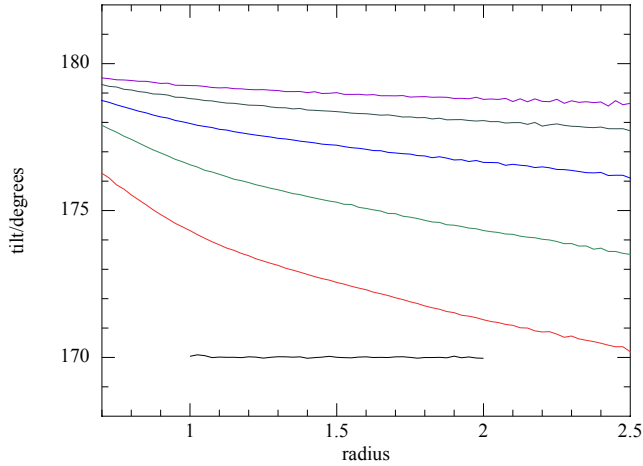
The implementation of accretion disc  $\alpha$ -viscosity (Shakura & Sunyaev, 1973) in PHANTOM is described in Lodato & Price (2010). Specifically, we use the ‘artificial viscosity for a disc’ described in Sec. 3.2.3 of Lodato & Price (2010), similar to earlier SPH accretion disc calculations (e.g. Murray, 1996). The main differences compared to standard SPH artificial viscosity are that the disc viscosity is applied to both approaching and receding particles and that no switches are used. The implementation used here differs slightly from Lodato & Price (2010) in that the  $\beta^{\text{AV}}$  term in the signal velocity is retained in order to prevent particle interpenetration. The disc viscosity in PHANTOM was extensively calibrated against the 1D thin  $\alpha$ -disc evolution in Lodato & Price (2010) (c.f. Fig. 4 in that paper) and the disc scale heights employed here are similar. We use  $\alpha^{\text{AV}} = 1$  and  $\beta^{\text{AV}} = 2$  which at the employed resolution (see below) corresponds to a physical viscosity  $\alpha_{\text{SS}} \approx 0.05$ . Note that the initial viscosity is slightly smaller than this, however during the simulation the disc spreads and particles are accreted and thus the spatial resolution decreases which increases the viscosity (see eq. 38 of Lodato & Price 2010).

### 3.1 Setup

The simulation has two equal mass sink particles representing the binary (of total mass unity in code units), on a circular orbit with separation 0.5 (in code units). Initially the circumbinary gas disc is flat and composed of 1 million SPH particles, in hydrostatic equilibrium from 1.0 to 2.0 in radius, and surface density distribution  $\Sigma \propto R^{-1}$ , set up by means of the usual Monte-Carlo technique. The vertical hydrostatic equilibrium corresponds to  $H/R = 0.05$  at  $R = 1$ . The equation of state for the gas is isothermal. The disc is initially tilted at  $170^\circ$  to the binary plane. Any gas which falls within a radius of 0.5 from the binary centre of mass is removed as it no longer has any effect of the alignment of the circumbinary disc. The disc mass is negligible in comparison to the binary mass and thus the gravitational back-reaction of the gas on the binary is not included.

### 3.2 Geometry

The binary is taken to orbit in the  $x$ - $y$  plane with binary angular momentum vector in the  $z$ -direction. We define the ‘tilt’ and the ‘twist’ of the disc with respect to the binary using Euler angles (e.g.



**Figure 1.** The tilt angle in the disc as a function of radius. This is plotted at six different times corresponding to (in units of the dynamical time at  $R = 1$ )  $t = 0$  (black),  $t = 500$  (red),  $t = 1000$  (green),  $t = 1500$  (blue),  $t = 2000$  (grey),  $t = 2500$  (purple). Initially the disc is flat with a global tilt of  $170^\circ$ . In time the disc spreads and counteraligns wrt the binary plane.

Bardeen & Petterson 1975; Pringle 1996), where the unit angular momentum vector in the disc is described at any radius by

$$\ell = (\cos \gamma \sin \beta, \sin \gamma \sin \beta, \cos \beta) \quad (7)$$

with  $\beta(R, t)$  the local angle of disc tilt with respect to the  $z$ -axis and  $\gamma(R, t)$  the local angle of disc twist measured from the  $x$ -axis.

### 3.3 Stable counteralignment

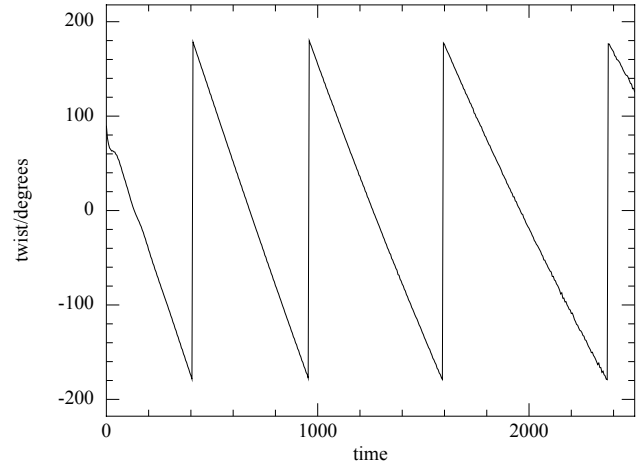
As predicted by NKP the dominant effect of the binary on the disc is to induce precessions in the gas orbits. As the radial range of the disc in this simulation is only a factor of two, the precession rate changes little between the inner and outer parts of the disc. However there is still a differential precession across the disc, which leads to a twist. This causes dissipation between rings of gas and so a small amplitude warp in the disc (cf. Fig. 1). The disc angular momentum vector precesses around the binary angular momentum vector for the duration of the simulation. In Fig. 2 the twist angle in the disc (at unit radius) is plotted against time. As the twist angle is defined between  $\pm 180^\circ$  the plot has a ‘saw-tooth’ shape, as the disc precesses until the twist is  $-180^\circ$  which is then equivalent to  $180^\circ$ .

Figs. 3 & 4 show the disc structures at various times. Fig. 3 shows the disc column density viewed face on, and Fig. 4 shows it edge on. In Fig. 3 we see the usual viscous spreading of the disc, and also the lack of any resonances in the disc. This is expected in a counterrotating circumbinary accretion disc (Papaloizou & Pringle 1977; Nixon et al. 2011). In Fig. 4 we can see the tilt of the disc. Initially the disc is tilted by  $170^\circ$ , but then precesses and counter-aligns with the binary plane.

I have also performed simulations (not illustrated) of a moderately eccentric binary ( $e = 0.3$ ) and an unequal mass ratio binary ( $M_2/M_1 = 0.5$ ) which both display the same behaviour.

## 4 CONCLUSIONS

In this paper I have shown that for realistic parameters a circumbinary disc must usually wholly co- or counter-align with a binary.



**Figure 2.** The twist angle (angle from  $x$ -axis to the line of nodes) at  $R = 1$  in the disc as a function of time. Initially the line of nodes is in the  $y$ -direction and hence the angle is  $90^\circ$ . The binary potential induces precession of the gas orbits. The twist is calculated between  $\pm 180^\circ$  which generates the ‘saw-tooth’ structure in the plot.

However for extreme mass ratios or high eccentricities the binary may be dominated by the disc angular momentum (cf. eq. 6). In this case evolution similar to Fig. 9 of Lodato & Pringle (2006) is expected with simultaneous co- and counter-alignment of the disc.

I have shown that a disc with an initial inclination of  $170^\circ$  to the binary plane stably counteraligns. Circumbinary discs, with  $J_d \ll J_b$  and an initial misalignment angle of  $> 90^\circ$  counteralign with respect to the binary. If the disc angular momentum is not negligible, King et al. (2005) showed that the condition for counteralignment is

$$\cos \theta < -\frac{J_d}{2J_b}. \quad (8)$$

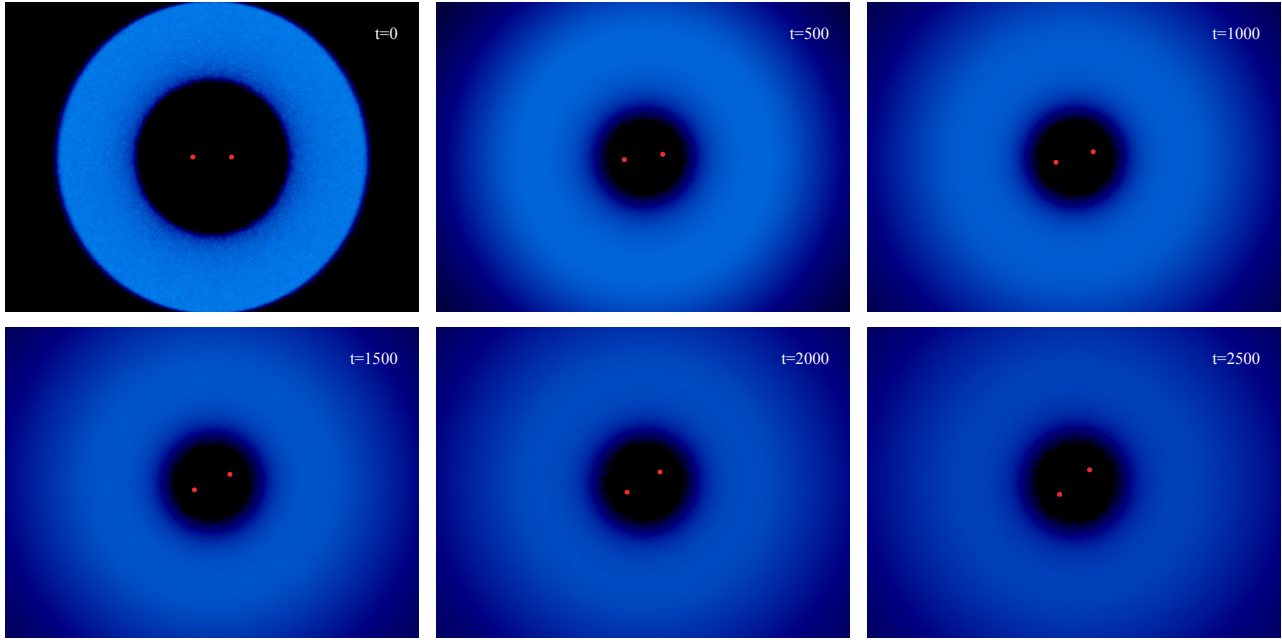
A counteraligned circumbinary disc is efficient at shrinking the binary as it directly absorbs negative angular momentum when capturing gas into circumprimary or circumsecondary discs (Nixon et al., 2011). This interaction increases the binary eccentricity. Nixon et al. (2011) show that the timescale to increase the eccentricity from zero to unity is  $\sim M_2/\dot{M}$  where  $\dot{M}$  is the mass inflow rate through the retrograde circumbinary disc. Once the eccentricity is high enough gravitational wave losses will drive the binary to coalescence.

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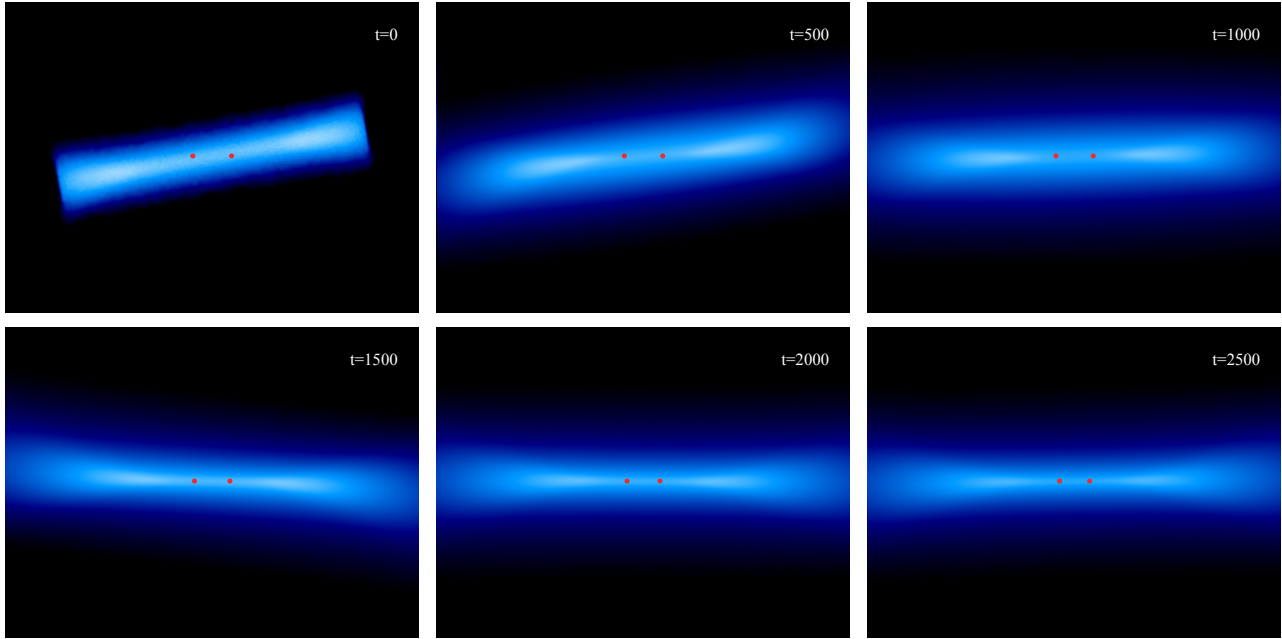
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**Figure 3.** Face-on column density rendering of the disc at various times in the simulation. Each component of the binary is represented by a red filled circle. Over time the disc spreads and precesses around the binary. The precession induces dissipation in the disc which aligns the disc with the binary plane.



**Figure 4.** Edge-on column density rendering of the disc at various times in the simulation. Each component of the binary is represented by a red filled circle. Over time the disc spreads and precesses around the binary. The precession induces dissipation in the disc which aligns the disc with the binary plane.

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